Astronomy & Astrophysics manuscript no. mkretlow-aa-mass29

The mass of (29) Amphitrite derived from its gravitational perturbations on (987) Wallia and (6904) McGill

M. Kretlow

Received 17 March 2005 / Withdrawn... never published.

ABSTRACT

The mass of (29) Amphitrite, an S-class asteroid in the main belt, was determined from its gravitational perturbations on the motion of (987) Wallia and (6904) McGill. For the mass of (29) Amphitrite a value of $(5.6 \pm 1.1) \times 10^{-12} M_{\odot}$ was obtained from the analysed observations of (987) Wallia and $(5.9 \pm 0.6) \times 10^{-12} M_{\odot}$ from the observations of (6904) McGill respectively. From these two independend results, the weighted mean value for the mass of (29) Amphitrite is $(5.9 \pm 0.6) \times 10^{-12} M_{\odot}$. Assuming an IRAS diameter of (212.2 ± 6.8) km this yields to a density $\rho = (2.4 \pm 0.3)\,\mathrm{g\,cm^{-3}}$.

Key words. celestial mechanics – methods: numerical – astrometry – minor planets, asteroids: individual: (29) Amphitrite

1. Introduction

Since the first determination of Vesta's mass by Hertz (1966) further values for about 30 asteroids were calculated so far by dynamical methods, some of them derived from the motion of their satellites rather then from perturbations on other asteroids (or planets like Mars). Overviews concerning latest mass determinations are given by Michalak (2000) and Hilton (2002).

Compilations of asteroid candidates for mass determinations and favourable encounters in the past and future were frequently published, the most recent ones by Hilton et al. (1996), Galád (2002), Galád & Gray (2002) and Fienga et al. (2003). The encounters of (29) Amphitrite with (987) Wallia and (6904) McGill were taken from the paper by Galád (2002) as possible candidates for the mass determination of (29) Amphitrite, though (6904) McGill was initially not suitable for this task since no pre-encounter observations were archived at the Minor Planet Center (MPC). However, as prediscovery observations were found and measured on digitized sky survey plates, (6904) McGill could be used for an independent mass determination.

(29) Amphitrite moves near the 3:1 resonance with Jupiter and was used by Schmadel (1986) to derive a new mass value of the Jupiter system. The IRAS diameter is $D = (212.2 \pm 6.8)$ km (Tedesco et al. 2004). (29) Amphitrite is classified as an S-type asteroid (Tholen 1989).

2. Dynamical model

The numerical integration was carried out by an n-body program developed by the author in the past years. The implemented integrator is a multi-step, variable order, predictor-corrector (PECE) method with self-adjusting stepsize (Shampine & Gordon 1975). The equations of motion include relativistic terms according to the Schwarzschild metric in isotropic coordinates (Sitarski 1983). The motion of the major planets (Mercury to Neptune) are not integrated within the program, rather their positions during the computation are read from the JPL DE405 ephemeris. In a similar manner the positions for the major perturbing asteroids (Table 1) which were used in the dynamical model are provided by external ephemerides which were generated before in separate runs.

The mass of the perturbing body was determined by means of a Least-Squares-Fit of the solve-for parameters to the observations by solving the equations of condition

$$P\Delta E = \Delta(\alpha, \delta). \qquad (1)$$

$P = \partial(\alpha, \delta)/\partial E$ is the matrix of the partial derivatives of the observed coordinates with respect to the corrections to the six rectangular initial values of the test asteroid and the mass of the perturbing body respectively. ΔE is the parameter vector of dimension seven with the corrections to the initial values (state vector) of the test asteroid and the correction to the mass of the perturbing body. Finally $\Delta(\alpha, \delta)$ is the vector of the residuals (O-C) in the observations of the perturbed body.

The partial derivatives P were not computed by numerical variation, but rather by integrating a set of seven differential equations together with the equations of motion of the test asteroid (Sitarski 1971, 1983).

3. Observations

The quantity, quality and distribution of the observations is crucial for the result and reliability of the mass determination. It

Send offprint requests to: M. Kretlow, e-mail: mike@obsnn.de

Table 1. Masses (adopted from Michalak (2000) & Michalak (2001)) and orbit references of the perturbing asteroids (and (29) Amphitrite) which are considered in the dynamical model.

Asteroid	Mass [$10^{-10}M_{\odot}$]	Orbit Reference
(1) Ceres	4.70	MPC 24219
(2) Pallas	1.21	MPC 24084
(4) Vesta	1.36	MPC 24219
(10) Hygiea	0.47	MPC 35055
(511) Davida	0.33	MPC 31002
(704) Interamnia	0.35	MPC 31002
(29) Amphitrite	–	MPO 57621

terms of quality it is desirable to bring all observational data and the dynamical model into a common and consistent reference system. To realize this approximatively the following reduction steps were applied.

For those observations, which are already converted by the MPC from FK4/B1950 to FK5/J2000 by a global rotation (Flag 'A' in column 15 of the current MPC observation format), zonal corrections FK5–FK4 were applied (Schwan 1988).

In a second step all observations prior to 1998 were transformed to the Hipparcos Catalog Reference System (HCRS) by a global rotation and subsequent correction for local differences between the Hipparcos and FK5 catalog (Mignard & Frœschlé 2000).

Observations from 1998 on were left untouched, despite the possibility that there might be some few observations after 1998, which were not reduced using a HCRS-linked star catalog[1]. The FASTT observations are made directly in the VLBI extragalactic reference frame (Stone 1997) and were not further processed in any way.

3.1. (987) Wallia

In total 767 observations for (987) Wallia were available, covering the overall timespan 1899 – 2004. However the useful timespan is 1922 – 2004, because the single discovery observation in 1899 exceeds the residual limits. The asteroid was not again observed until 1922. Most of the early observations prior to 1960 are coarse or semi-accurate and were rejected right from the beginning, thus only a fractional part of the observations cover the time before 1960. Re-reducing this old plates (25 observations 1922 – 1958) would in principle improve the calculations. Since the encounter with (29) Amphitrite was in 1994, these very first observations (before 1960) do not have major importance to establish a reliable pre-encounter orbit of (987) Wallia.

3.2. (6904) McGill

The asteroid was discovered in 1990 and thus no pre-encounter observations were initially available. A search in the Digitized Sky Surveys (DSS) revealed pre-discovery images on plates taken in 1951, 1955 (POSS-I) and 1982 (SERC-J). They were reduced using the UNSO-B1.0 star catalog and thus are already in a HCRS-linked reference system (Table 6). Only the 1990 observations of the discovery opposition are still provided in the FK4-to-FK5 rotated system and therefore they were locally corrected as described before. All subsequent observations were already made in the FK5-system. This observational data set is very consistent and of good accuracy – only 5% of 367 observations were rejected at last.

[1] Unfortunately the current MPC 80-column archive format does not indicate the star catalog which was used for the reduction of an individual observation.

4. Calculations

Table 2. Encounters to (29) Amphitrite (sorted by increasing deflection angle) and of the test asteroids to other asteroids closer than 0.02 AU for the time 1900 – 2015. The last three columns give the minimal encounter distance d, the relative encounter velocity v and the deflection angle θ of the test asteroid (column 2) with respect to (29) Amphitrite (with mass adopted from this work).

Asteroid number 1	Asteroid number 2	Date y/m/d	d [AU]	v [km/s]	θ [arcs]
(29)	(1433)	1942/12/12	0.01027	2.06	0.05
(29)	(14809)	1986/01/17	0.00539	2.47	0.07
(29)	(987)	1994/03/03	0.00245	3.19	0.09
(29)	(6904)	1985/06/15	0.00933	0.57	0.71
(987)	–	–	–	–	–
(6904)	(411)	1923/10/24	0.00938	6.78	–
(6904)	(739)	1976/06/29	0.01156	9.61	–

Both test asteroids were also checked for any close encounters with other asteroids which may have influenced their orbits significantly in the past (Table 2). The deflection angle θ This was done by integrating their orbits over the period 1900 – 2015 each time together with one asteroid given in the Lowell database of orbital elements (`astorb.dat`) with a diameter $D > 50$ km (either IRAS diameter given in the file or estimated from the absolute magnitude). (987) Wallia had no disturbing encounters to other asteroids in this period whereas (6904) McGill had two encounters which should be noticed. The encounter with (411) Xanthe was almost 30 years before the first available observation in 1951 and was much weaker compared to the encounter with (29) Amphitrite in 1985. The same holds for the encounter with (739) Mandeville in 1976 which was even weaker, since the change in heliocentrical velocity of the perturbed (test) asteroid is $\Delta v \sim 1/r_0 v_0$, where r_0 is the minimum distance and v_0 is the corresponding relative velocity of the encounter.

For (29) Amphitrite itself it was verified whether there were close encounters to other asteroids, either to be used for the mass determination or to check if (29) Amphitrite had close encounters with other asteroids of similar size and mass which may have influenced its orbit. The encounter between (29) Amphitrite and (1433) Geramtina in 1942 was found to be promising for mass determination beside the encounters with (987) Wallia and (6904) McGill. However, the impact parameters was probably still too large and no reliable mass could be derived from this test asteroid. In fact the encounters with

(987) Wallia and (6904) McGill are actual the most suitable ones for the determination of (29) Amphitrite's mass.

The IRAS diameter for (987) Wallia is estimated to be about 44 km, the taxonomic class is unknown. Assuming, that it is also an S-type asteroid, the mass of this test asteroid is more than 100 times smaller than the mass of the perturbing asteroid (29) Amphitrite. Thus the perturbations on (29) Amphitrite by (6904) McGill (diameter of about 5 km is estimated from the absolute magnitude) should be even much smaller.

The calculations were done independently for each test asteroid. Their initial orbital elements were taken from the `mpcorb.dat` file provided by the Minor Planet Center (MPC). The accepted final mass of (29) Amphitrite is the weighted mean of both individual results. The weighting (and rejection) of the observations was done iteratively. For each iteration, several orbit correction runs were made until convergence (i.e. the RMS value of one coordinate was stable within 0.001 arcs). Then all observations with residuals exceeding 3σ were rejected and the next iteration was started. This was repeated until no more observations were rejected and the RMS value of the computation converged to a final value. Neither individual weights in (α, δ) nor dynamical weights corresponding to the residual of each observation were applied.

5. Results and Discussion

Table 3 summarizes the results. While the mass estimates for (29) Amphitrite obtained from both test asteroids are very similar, the formal uncertainty of this value for (987) Wallia is much larger. Probably this is caused by the stronger correlation between the mass and the rectangular initial values compared to (6904) McGill. The largest correlation coefficient for (987) Wallia is 0.798 (Table 4) while for (6904) McGill it is just 0.357 (Table 5). It should also be noticed that the deflection angle θ is about eight times larger for (6904) McGill then for (987) Wallia (Table 2). In particular for (987) Wallia, the obtained mass values show also some scatter depending whether or not corrections for the different reference systems were taken into account as described in Section 3. If no corrections were made to the observations of (987) Wallia at all, a mass $M_{29} = (6.7 \pm 1.6) \times 10^{-12} M_{\odot}$ was found. The corresponding density $\rho = (2.7 \pm 0.7)$ agrees well with the expected value $\rho = 2.7\,\mathrm{g\,cm^{-3}}$ (Britt et al. 2002) for an S-type asteroid. For (6904) McGill this scatter is clearly smaller because the majority of observations were originally reduced in FK5 or HCRS linked systems.

Table 3. Results for the mass of (29) Amphitrite. The mean bulk density is calculated using the given IRAS diameter (212.2 ± 6.8) km.

Asteroid	Mass M_{29} [$10^{-12} M_{\odot}$]	Density [$\mathrm{g\,cm^{-3}}$]	RMS [arcs]	Observ. acc. / total
(987) Wallia	6.1 ± 1.7	2.4 ± 0.7	0.461	541 / 683
(6904) McGill	5.9 ± 0.6	2.4 ± 0.3	0.511	349 / 367
Weighted mean:	5.9 ± 0.6	2.4 ± 0.3		

Table 4. Correlation matrix for the test asteroid (987) Wallia.

x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	M_{29}
+1.000	-0.704	-0.313	+0.989	+0.953	+0.759	-0.282
	+1.000	-0.368	-0.660	-0.599	-0.841	+0.216
		+1.000	-0.293	-0.456	+0.162	+0.077
			+1.000	+0.938	+0.742	-0.300
				+1.000	+0.597	-0.357
					+1.000	-0.293
						+1.000

Table 5. Correlation matrix for the test asteroid (6904) McGill.

x	y	z	$\dot{x}$	$\dot{y}$	$\dot{z}$	M_{29}
+1.000	-0.704	-0.313	+0.989	+0.953	+0.759	-0.282
	+1.000	-0.368	-0.660	-0.599	-0.841	+0.216
		+1.000	-0.293	-0.456	+0.162	+0.077
			+1.000	+0.938	+0.742	-0.300
				+1.000	+0.597	-0.357
					+1.000	-0.293
						+1.000

6. Conclusions

The mass of (29) Amphitrite was estimated from the analysis of the orbital motion of the asteroids (987) Wallia and (6904) McGill. A value $M_{29} = (5.9 \pm 0.6) \times 10^{-12} M_{\odot}$ was obtained as a weighted mean of both individual results. Assuming an IRAS diameter $D = (212.2 \pm 6.8)$ km, a mean bulk density of $\rho = (2.4 \pm 0.3)\,\mathrm{g\,cm^{-3}}$ is derived. This value is within the expected density range for S-class asteroids.

Acknowledgements. I am grateful to Dr. H. Boehnhardt (MPS, Lindau) as well as to Dr. L. D. Schmadel and Prof. Dr. J. Schubart (Astronomisches Recheninstitut (ARI), Heidelberg) for their valuable remarks and suggestions. I thank R. M. Stoss (DANEOPS Team) for measuring the pre-discovery observations (1951–1982) of (6904) McGill. I also thank Dr. M. Read and Dr. S. Tritton (IfA, University of Edinburgh) for kindly inspecting the 1985 UKST plates. All institutions and persons involved in the preparation of the Digitized Sky Survey (DSS) and the underlying observations are appreciated (see `http://archive.stsci.edu/dss/acknowledging.html`).

References

Britt, D.T., Yeomans, D., Housen, K., et al. 2002, in Asteroids III, eds. W. F. Bottke, A. Cellino, P. Paolicchi, et al. (The Univ. of Arizona Press, Tucson), 488.
Fienga, A., Bange, J.-F., Bec-Borsenberger, A., & Thuillot, W. 2003, A&A406, 751.
Galád, A., 2001, A&A 391, 121.
Galád, A., Gray, B. 2002, A&A 391, 121.
Hertz, H.G., 1966, IAU Circular No. 1983.
Hilton, J.L., Seidelmann, P.K., Middour, J. 1996, AJ 112, 2319.
Hilton, J.L. 2002, in Asteroids III, eds. W. F. Bottke, A. Cellino, P. Paolicchi, et al. (The Univ. of Arizona Press, Tucson), 105.
Michalak, G. 2000, A&A 360, 363.
Michalak, G. 2001, A&A 374, 703.
Mignard, F., Frœschlé 2000, A&A 354, 732.
Schmadel, L. D. 1986, Astronomische Nachrichten 307, 363.

Table 6. Pre-discovery observations of (6904) McGill given in the 80-column MPC format.

```
06904         h1951 08 11.42639 00 06 59.69 +00 51 47.4                      261
06904         h1951 08 11.43333 00 06 59.55 +00 51 47.7                      261
06904         h1951 08 11.46805 00 06 59.02 +00 51 49.4                      261
06904         h1955 11 11.13611 00 00 58.25 +05 51 33.3                      261
06904         h1955 11 11.14167 00 00 58.16 +05 51 33.2                      261
06904         h1955 11 11.17639 00 00 57.84 +05 51 32.4                      261
06904         h1982 07 16.67712 22 16 09.66 -15 08 45.6                      260
06904         h1982 07 16.71879 22 16 08.54 -15 08 49.5                      260
```


Schwan, H. 1988, A&A 198, 363. (Algorithms and FORTRAN code provided by ARI Heidelberg).

Shampine, L.F. and Gordon, M.K., 1975, Computer Solution of Ordinary Differential Equations: the Initial Value Problem, W. H. Freeman and Co., San Francisco.

Sitarski, G. 1971, Acta Astron. 21, 87.

Sitarski, G. 1983, Acta Astron. 33, 295.

Stone, R.C. 1997, AJ 113, 2317.

Tedesco, E.F., P.V. Noah, M. Noah, et al., IRAS Minor Planet Survey. IRAS-A-FPA-3-RDR-IMPS-V6.0. NASA Planetary Data System, 2004.

Tholen, D.J. 1989, in Asteroids II, eds. R.P.Binzel, T.Gehrels, M.S.Matthews (The Univ. of Arizona Press, Tucson), 1141.